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The Limits of Custodial Symmetry [a]

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We introduce a toy model implementing the proposal of using a custodial symmetry to protect the $Zb_L\bar{b}_L$ coupling from large corrections. This "doublet-extended standard model" adds a weak doublet of fermions (including a heavy partner of the top quark) to the particle content of the standard model in order to implement an $O(4)\times U(1)_X \sim SU(2)_L\times SU(2)_R\times P_{LR}\times U(1)_X$ symmetry that protects the $Zb_L\bar{b}_L$ coupling. This symmetry is softly broken to the gauged $SU(2)_L \times U(1)_Y$ electroweak symmetry by a Dirac mass M for the new doublet; adjusting the value of M allows us to explore the range of possibilities between the $O(4)$-symmetric ($M \to 0$) and standard-model-like ($M \to \infty$) limits.

1 Introduction

Agashe [2] et al. have shown that the constraints on beyond the standard model physics related to the $Zb_L\bar{b}_L$ coupling can, in principle, be loosened if the global $SU(2)_L \times SU(2)_R$ symmetry of the electroweak symmetry breaking sector is actually a subgroup of a larger global symmetry of both the symmetry breaking and top quark mass generating sectors of the theory. In particular, they propose that these interactions preserve an $O(4) \sim SU(2)_L \times SU(2)_R \times P_{LR}$ symmetry, where P_{LR} is a parity interchanging $L \leftrightarrow R$. The $O(4)$ symmetry is then spontaneously broken to $O(3) \sim SU(2)_V \times P_{LR}$, breaking the elecroweak interactions but protecting g_{Lb} from radiative corrections, so long as the left-handed bottom quark is a P_{LR} eigenstate.

In this talk we report on the construction of the simplest $O(4)$-symmetric extension of the SM.[1], the doublet-extended standard model or DESM.

1.1 The Model

We extend the global $SU(2)_L \times SU(2)_R$ symmetry of the Higgs sector of the SM to an $O(4) \times U(1)_X \sim SU(2)_L \times SU(2)_R \times P_{LR} \times U(1)_X$ for both the symmetry breaking and top quark mass generating sectors of the theory. As usual, only the electroweak subgroup, $SU(2)_L \times U(1)_Y$, of this global symmetry is gauged; our model does not include additional electroweak gauge bosons. The global $O(4)$ spontaneously breaks to $O(3) \sim SU(2)_V \times P_{LR}$ which will protect g_{Lb} from radiative corrections,[2] provided that the left-handed bottom quark is a parity eigenstate: $P_{LR} b_L = \pm b_L$. The additional global $U(1)_X$ group is included to ensure that the light t and b eigenstates, the ordinary top and bottom quarks, obtain the correct hypercharges.

We therefore introduce a new doublet of fermions $\Psi \equiv (\Omega, T')$. The left-handed component,

[a] Speaker at conference: R. Sekhar Chivukula. This report is a shortened version of previously published work.[1]

Table 1: Charges of the fermions under the various symmetry groups in the model. Note that, as discussed in the text, other T^3_R and Q_X assignments for the Ω_R and T'_R states are possible.

	t'_L	b_L	Ω_L	T'_L	t'_R	b_R	Ω_R	T'_R
T^3_L	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
T^3_R	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	-1	0	0
Q	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{5}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{5}{3}$	$\frac{2}{3}$
Y	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{7}{6}$	$\frac{7}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{7}{6}$	$\frac{7}{6}$
Q_X	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{7}{6}$	$\frac{7}{6}$

Ψ_L joins with the top-bottom doublet $q_L \equiv (t'_L, b_L)$ to form an $O(4) \times U(1)_X$ multiplet

$$\mathcal{Q}_L = \begin{pmatrix} t'_L & \Omega_L \\ b_L & T'_L \end{pmatrix} \equiv \begin{pmatrix} q_L & \Psi_L \end{pmatrix} \ , \tag{1}$$

which transforms as a $(2, 2^*)_{2/3}$ under $SU(2)_L \times SU(2)_R \times U(1)_X$. The parity operation P_{LR}, which exchanges the $SU(2)_L$ and $SU(2)_R$ transformation properties of the fields, acts on $\mathcal{Q}_L$ as:

$$P_{LR}\mathcal{Q}_L = -\left[(i\sigma_2)\,\mathcal{Q}_L\,(i\sigma_2)\right]^T = \begin{pmatrix} T'_L & -\Omega_L \\ -b_L & t'_L \end{pmatrix} \tag{2}$$

exchanging the diagonal components, while reversing the signs of the off-diagonal components. The t' and T' states mix to form mass eigenstates corresponding to the top quark (t) and a heavy partner (T).

We assign the minimal right-handed fermions charges that accord with the symmetry-breaking pattern we envision: the top and bottom quarks will receive mass via Yukawa terms that respect the full $O(4) \times U(1)_X$ symmetry, while the exotic states will have a dimension-three mass term that explicitly breaks the large symmetry to $SU(2)_L \times U(1)$. The charges of all the fermions are listed in Table 1.

Now, let us describe the symmetry-breaking pattern and fermion mass terms explicitly. Spontaneous electroweak symmetry breaking proceeds through a Higgs multiplet that transforms as a $(2, 2^*)_0$ under $SU(2)_L \times SU(2)_R \times U(1)_X$:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h + i\phi^0 & i\sqrt{2}\,\phi^+ \\ i\sqrt{2}\,\phi^- & v + h - i\phi^0 \end{pmatrix} \ . \tag{3}$$

Again, the parity operator P_{LR} exchanges the diagonal fields and reverses the signs of the off-diagonal elements. When the Higgs acquires a vacuum expectation value, the longitudinal W and Z bosons acquire mass and a single Higgs boson remains in the low-energy spectrum. The Higgs multiplet has an $O(4) \times U(1)_X$ symmetric Yukawa interaction with the top quark:

$$\mathcal{L}_{\text{Yukawa}} = -\lambda_t \mathrm{Tr}\left(\overline{\mathcal{Q}}_L \cdot \Phi\right) t'_R \ + \text{h.c.} \ . \tag{4}$$

that contributes to generating a top quark mass.[b]

Next we break the full $O(4) \times U(1)_X$ symmetry to its electroweak subgroup. We do so first by gauging $SU(2)_L \times U(1)_Y$. In addition, we wish to preserve the $O(4)$ symmetry of the top quark mass generating sector in all dimension-4 terms, but break it softly by introducing a dimension-3 Dirac mass term for Ψ,

$$\mathcal{L}_{\text{mass}} = -M\ \bar{\Psi}_L \cdot \Psi_R + h.c. \tag{5}$$

[b] Here we neglect m_b and any other Yukawa interactions.

that explicitly breaks the global symmetry to $SU(2)_L \times U(1)_Y$. Note that we therefore expect that any flavor-dependent radiative corrections to the $Zb_L\bar{b}_L$ coupling will vanish in the limit $M \to 0$, as the protective parity symmetry is restored; alternatively, as $M \to \infty$, the larger symmetry is pushed off to such high energies that the resulting theory looks more and more like the SM.

1.2 Mass Matrices and Eigenstates

When the Higgs multiplet acquires a vacuum expectation value and breaks the electroweak symmetry, masses are generated for the top quark, its heavy partner T and the exotic fermion Ω through the mass matrix:

$$\mathcal{L}_{\text{mass}} = -\begin{pmatrix} t'_L & T'_L \end{pmatrix} \begin{pmatrix} m & 0 \\ m & M \end{pmatrix} \begin{pmatrix} t'_R \\ T'_R \end{pmatrix} - M\bar{\Omega}_L\Omega_R + \text{h.c} \,, \tag{6}$$

where

$$m = \frac{\lambda_t v}{\sqrt{2}} \,. \tag{7}$$

Diagonalizing the top quark mass matrix yields mass eigenstates t (corresponding to the SM top quark) and T (a heavy partner quark), with corresponding eigenvalues

$$m_t^2 = \frac{1}{2}\left[1 - \sqrt{1+\frac{4m^4}{M^4}}\right] M^2 + m^2 \,, \qquad m_T^2 = \frac{1}{2}\left[1 + \sqrt{1+\frac{4m^4}{M^4}}\right] M^2 + m^2 \,. \tag{8}$$

The mass eigenstates are related to the original gauge eigenstates through the rotations whose mixing angles are given by

$$\sin\theta_R = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{1-2m^2/M^2}{\sqrt{1+4m^4/M^4}}} \,, \quad \sin\theta_L = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{1}{\sqrt{1+4m^4/M^4}}} \,. \tag{9}$$

From these equations the decoupling limit $M \to \infty$ is evident: m_t approaches its SM value as in Eq. (7), the $t-T$ mixing goes to zero, and T becomes degenerate with Ω. Conversely, in the limit $M \to 0$, the full $O(4) \times U(1)_X$ symmetry is restored and only the combination $T'_L + t'_L$ couples to t_R with mass m. For phenomenological discussion, it will be convenient to fix m_t at its experimental value and express the other masses in terms of m_t and the ratio $\mu \equiv M/m$.

2 δg_{Lb}, αS, and αT

We now display the value of the $Z\bar{b}_L b$ coupling, g_{Lb}, in our model [1] (as a function of μ for fixed m_t), and compare with the values given by experiment and the SM, as illustrated in Fig. (1). The (solid blue) curve shows how g_{Lb} varies with μ in our model; we required g_{Lb} to match the SM value with $m_t = 172$ GeV and $v = 246$ GeV as $\mu \to \infty$. We see that g_{Lb} in our model is slightly more negative than (i.e. slightly farther from the experimental value than) the SM value for $\mu > 1$, agrees with the SM value for $\mu = 1$, and comes within $\pm 1\sigma$ of the experimental value only for $\mu < 1$. Given the shortcomings of the small-μ limit, this is disappointing.

Furthermore, in Figure 2 we show the DESM predictions [1] for the oblique parameters [5,6,7] $[\alpha S^{th}(\mu), \alpha T^{th}(\mu)]$ using $m_h = 117$ GeV, and illustrating the successive mass-ratio values $\mu =$ 3, 4, ..., 20, ∞; the point $\mu = \infty$ corresponds to the SM limit of the DESM and therefore lies at the origin of the αS - αT plane. From this figure, we observe directly that the 95%CL lower limit on μ for $m_h = 115$ GeV is about 20, while for any larger value of m_h the DESM with $\mu \leq 20$ is excluded at 95%CL. In other words, the fact that a heavier m_h tends to worsen the fit of

Figure 1: The solid (blue) curve shows the DESM model's prediction for g_{Lb} The thick horizontal line corresponds to $g_{Lb}^{ex} = -0.4182$, while the two horizontal upper and lower solid lines bordering the shaded band correspond to the $\pm 1\sigma$ deviations [4]. The SM prediction is given by the dashed horizontal line. The leading-log contribution is shown by the dotted curve.

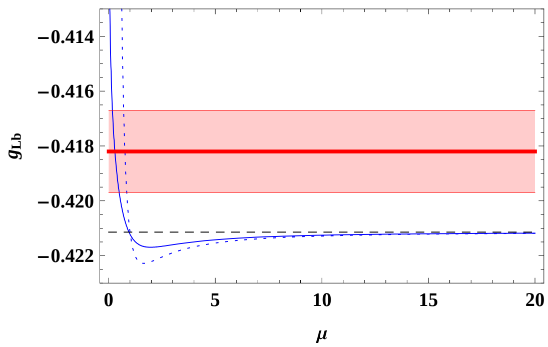
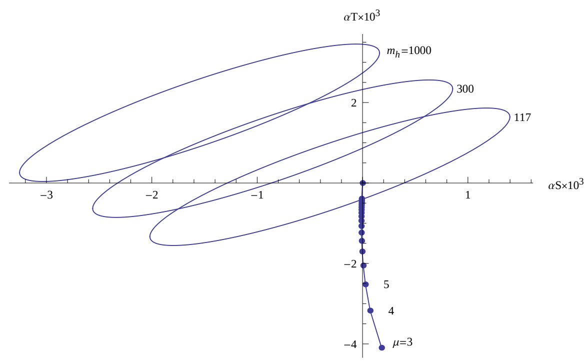


Figure 2: The dots represent the theoretical predictions of the DESM (with m_h set to the reference value 115 GeV), showing how the values of αS and αT change as μ successively takes on the values 3, 4, 5, ..., 20, ∞. The three ellipses enclose the 95%CL regions of the αS - αT plane for the fit to the experimental data performed in [3]; they correspond to Higgs boson mass values of $m_h = 115\,\text{GeV}$, $300\,\text{GeV}$, and $1\,\text{TeV}$. Comparing the theoretical curve with the ellipses shows that the minimum allowed value of μ is 20, for $m_h = 117$ GeV.



even the SM ($\mu \to \infty$) to the electroweak data is exacerbated by the new physics contributions within the DESM. The bound $\mu \geq 20$ corresponding to a DESM with a 115 GeV Higgs boson also implies, at 95%CL, that $m_T \geq \mu\, m_t \cong 3.4$ TeV, so that the heavy partners of the top quark would likely be too heavy for detection at LHC.

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